

# Lattice Effective Actions and Light-Quark Confinement

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The positive-plaquette Manton action at weak coupling is a reasonable action for short-distance phenomena. We propose an iterative scheme for evolving this action into an effective action for longer distance scales. We report on the first step of this scheme in which we have measured “blocked” Creutz ratios with lattice spacing  $2a$  at  $\beta = 16$  on a  $32^4$  lattice and have searched for an effective action that yields the same ratios on a  $16^4$  lattice.

We also suggest a mechanism for quark confinement that relies upon the lightness of the  $u$  and  $d$  quarks and formulate a way of testing it in lattice simulations of  $QCD$ .

## 1. INTRODUCTION

Most lattice simulations are guided by Wilson’s action [1] in which the matrices of a compact group replace the fields of the continuum theory. In these simulations charged particles are confined at strong coupling whether the gauge group is abelian [2] or non-abelian [3,4]. There have been a few lattice simulations in which the basic variables are fields. Some of these non-compact simulations have no exact gauge symmetry and have shown no sign of confinement for either abelian or non-abelian theories [5,6]. In others gauge invariance was partially restored by the imposition of random gauge transformations, but it is unclear whether the resulting weak confinement signal [7] was due to the decorrelations produced by the noise of the random gauge transformations or to the attractive forces of the gauge bosons. Some very interesting simulations [8,9] possess an exact lattice gauge symmetry and display confinement for  $SU(2)$  and  $SU(3)$  but not for  $U(1)$  above  $\beta = 0.5$ . The gauge fields of these simulations, however, are not hermitian. In all simulations, whether compact or non-compact, confinement has appeared only when accompanied by significant lattice artifacts.

At small coupling the Wilson action and the Manton action [10] are plausible fundamental actions suitable for the description of gauge fields at short distances. But due to their artifacts, these actions cannot be fundamental actions at

moderate or strong coupling. At such couplings these actions might be effective actions suitable for describing gauge fields at longer distances of the order of a fraction of a fermi. Yet it is also possible, as Gribov has suggested [11], that the mechanism of confinement depends in a crucial way upon the lightness of the  $u$  and  $d$  quarks. In this paper we shall discuss these two possibilities and shall suggest ways of testing them.

## 2. EFFECTIVE ACTIONS

At very weak coupling, the best single-plaquette compact action is probably the one proposed by Manton, which for an  $SU(2)$  plaquette  $P$  is

$$S = \frac{\beta}{2} \arccos^2 \left( \frac{1}{2} \text{Tr}(P) \right). \quad (1)$$

When the plaquette is close to the identity, this action is proportional to the square of the compact field strength and so resembles the classical action. At  $\beta < 16$  it is useful to reject plaquettes that have negative trace.

To determine what effective action this fundamental, short-distance action evolves into at longer distances, we have begun a simulation of  $SU(2)$  gauge theory at  $\beta = 16$  on a  $32^4$  lattice. On this lattice our action is Manton’s but with plaquettes of negative trace assigned infinite action. We measure Wilson loops  $W(i, j)$  up to  $12 \times 12$  and compute both ordinary Creutz ra-

tios and blocked Creutz ratios

$$\chi(i, j, 2) = -\log \left( \frac{W(i, j)W(i-2, j-2)}{W(i-2, j)W(i, j-2)} \right) \quad (2)$$

in which the lattice spacing is  $2a$  instead of  $a$  and both  $i$  and  $j$  are even. Our strategy is to experiment with arbitrary actions on  $16^4$  lattices so as to find one whose ordinary Creutz ratios are equal to the blocked Creutz ratios on the  $32^4$  lattice. If we find such an effective action, then we plan to use it on a new  $32^4$  lattice and to measure both ordinary and blocked Creutz ratios on that lattice. The next step would be to search for a second effective action that on a second  $16^4$  lattice gives ordinary Creutz ratios that are equal to the blocked ones of the second  $32^4$  lattice.

The principal problem with this scheme is that large Wilson loops converge very slowly at weak coupling. In the first step of our implementation of this procedure at  $\beta = 16$ , we performed 392,000 thermalization sweeps on a  $16^4$  lattice and then used 16 clones of this lattice as our initial  $32^4$  lattice. After 20,000 thermalization sweeps on the  $32^4$  lattice, the blocked Creutz ratios  $\chi(4, 4, 2) = 0.01531(6)$ ,  $\chi(4, 6, 2) = 0.01035(6)$ , and  $\chi(4, 8, 2) = 0.00950(5)$  of the smaller loops may be close to converging. But the blocked ratios  $\chi(6, 6, 2) = 0.00465(4)$ ,  $\chi(6, 8, 2) = 0.00354(4)$ , and  $\chi(8, 8, 2) = 0.00227(5)$  of the larger loops are still trending upward. The errors quoted for these large loops are purely statistical and do not contain the systematic error of the secular drift. We have searched for an effective action that would yield these ratios on a  $16^4$  lattice; the closest so far is one with  $\beta = 15$ .

One may iterate this scheme provided one can find a new effective action at each step. In this case the successive effective actions eventually should evolve into a suitable long-distance effective action. This action might indeed be turn out to be Wilson's action at moderate coupling. After a few iterations, however, there may be no available single-plaquette effective action that yields Creutz ratios on the  $16^4$  lattice that are approximately equal to the blocked ratios of the preceding  $32^4$  lattice. If the scheme hits a wall in this way, then present lattice methods may be meaningless, and the reason for confinement may be

the lightness of the lighter quarks.

### 3. LIGHT-QUARK CONFINEMENT

We now wish to propose a simple dynamical mechanism for quark confinement which implements Gribov's idea [11] about the possible importance of light quarks. We also shall suggest a way of testing this mechanism.

The hamiltonian of  $QCD$  describing the interaction of the light  $u$  and  $d$  quarks with the  $SU(3)$  gauge fields  $A_\mu^b$  contains the pieces

$$V = -ig \bar{u} \not{A}_b^b u - ig \bar{d} \not{A}_b^b d \quad (3)$$

and

$$M = m_u \bar{u} u + m_d \bar{d} d. \quad (4)$$

Because  $V$  does not have a definite sign, it seems possible that in the physical vacuum of  $QCD$  the light quarks  $u$  and  $d$  might condense in ways that are correlated with the fluctuating gauge field  $A_\mu^b$ . In this picture the physical vacuum is represented by a functional integral

$$|\Omega\rangle = \int DA_\mu^b \Psi(A_\mu^b, u, d) |A_\mu^b, u, d\rangle \quad (5)$$

over a state  $|A_\mu^b, u, d\rangle$  in which the gluon variables form something like a coherent state with mean value  $A_\mu^b(x)$  and in which the quark variables  $u$  and  $d$  are correlated with the field  $A_\mu^b(x)$  in such a way that the mean value of the interaction  $V$  is large and negative

$$\langle \Omega | V | \Omega \rangle < 0. \quad (6)$$

Because the  $u$  and  $d$  quarks are light, the effect  $\langle \Omega | M | \Omega \rangle$  of their masses is small, and the mean value of  $V + M$  is large and negative

$$\langle \Omega | V + M | \Omega \rangle < 0. \quad (7)$$

In such a physical vacuum, pairs of up and down quarks from the condensate can convert pairs of quarks created in the debris of hadronic collisions into mesons.

Inasmuch as the proposed confinement mechanism is intrinsically non-perturbative, it is probably necessary to test it in a lattice simulation.

The signal would be a drop in the euclidean action  $S_q[U, u, d, \bar{u}, \bar{d}]$  of the quarks with the onset of confinement, as indicated by the Creutz ratios of the Wilson loops. We intend to use the QCDF90 codes [12] to perform this test.

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